Activity Title: "Freethrows win games"

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Abstract: Students use sports averages as data for estimating probabilities and making predictions about the outcomes of game situations. They use tree diagrams to organize the information needed for their probably calculations.

Objective: Students will investigate methods of calculating probability of compound events.

Grade Level: 9^{th}, 10^{th} grade

Materials: Paper, pencil, calculators, overhead projector and/or SmartBoard, copy of activity sheet for each student.

Class Time: This lesson was designed to be completed in one 50-minute class period although further discussion on the enrichment problem may be necessary if the students attempt the problem as homework or outside of class.

For the Instructor:

Pass out the activity sheet and discuss the context. Make sure students note that sports data are meticulously gathered on many details of a player's performance. Discuss why this is the case. A player's statistics matter in strategic decisions opponents make and in salary negotiations.

Assessment problems for Probability of Compound Events:

1. Steve Nash plays for the Phoenix Suns in the NBA. He is a very good freethrow shooter who makes 91% of his freethrows. If Steve Nash is fouled and is attempting 2 freethrows, what is the probability that he will make at least one of the two freethrows? Justify your answer by using a tree diagram or an area model.
   1 - .09*.09 = .9919

2. A bag of marbles contains 5 red, 3 blue, and 2 yellow marbles. Two marbles are to be selected without replacing the marble after the first choice. Find the probability of drawing both yellow marbles from the bag. Justify your answer.
   2/10 * 1/9 = 1/45 or.0222

FREETHROWS WIN GAMES !!

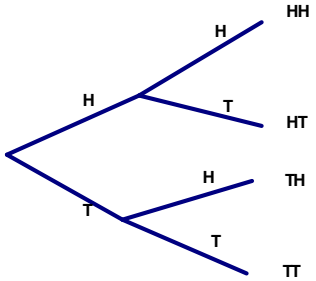
Shaquille O'Neal is one of the greatest basketball players in the NBA. He has won 4 NBA titles with the Los Angeles Lakers and the Miami Heat. He's listed in the NBA top 50 all time greatest list (and he's still playing!) and is currently ranked number 9 in SLAM Magazine's Top 75 NBA Players of all time. At 7'1" and 315 pounds, he can dominate the game around the basket. Unfortunately, Shaq is not a good freethrow shooter. His career freethrow is 52.5%, which is among the lowest in the NBA. Many teams would resort to fouling Shaq anytime he got the ball in order to stop him from scoring. In one game versus the Utah Jazz, Shaq's team, the Miami Heat was behind by one point when Shaq was fouled shooting a shot as the time ran out. Since, no time remained in the game, Shaq's 2 freethrow shots would determine the outcome of the game or whether the game went into overtime.

In pairs or small groups: List the possible outcomes for this situation in order from most likely to least likely.

After discussion on the situation we will use probability to study the likelihood of each of these possible outcomes. During this lesson, we will calculate which option is the most likely to occur.

Basic Probability of compound events

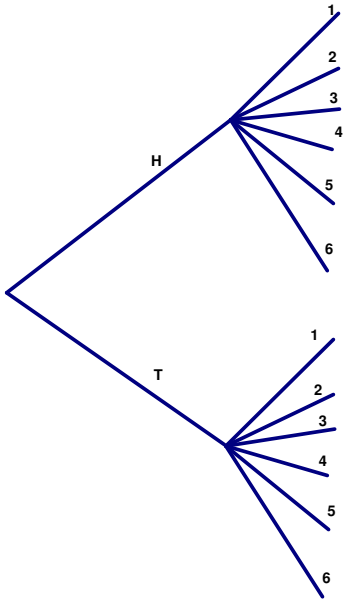
- Consider an easier example of flipping a coin. Obviously, there are two equally likely outcomes: Heads or Tails. If we flip the coins twice, now how many equally likely outcomes are there? (Four) These outcomes can be visualized with a tree diagram.



What is the probability of each of these outcomes? (Since each is equally likely, the probability should be ¼ or 25%)
Is it possible to get that same answer a different way? (Lead the students to the idea of multiplying ½ times ½)

- Extend this problem to discussing the probability of getting 3, 4, 5, 6, Heads in a row when flipping a coin. Discuss the fact that it is much easier to flip one Head than to try to flip 4 Heads in a row. This also leads to the idea of multiplying probabilities of independent events to obtain the probability of compound events.

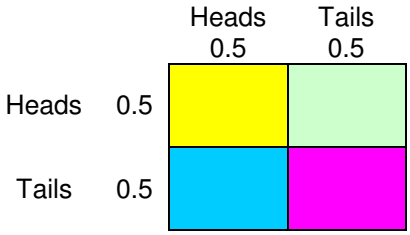
- What happens if we change the problem to flipping one coin and rolling one six-sided die? Draw a tree diagram for this situation.

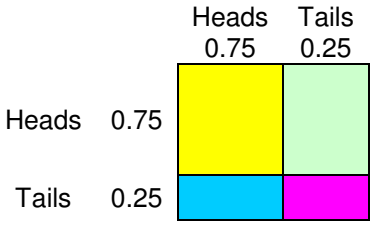


- How many equally likely outcomes are there? (12) How can you arrive at that answer from the Multiplication Counting Principal? (multiplying 2 * 6)
- What would the probability be of flipping a Heads and then rolling a 1? What about flipping a Tails and then rolling a 6? How can you use multiplication to come up with these answers? (1/2 * 1/6 . Lead the students to multiplying the two individual probabilities without using the tree diagram.)
- What do you think the probability would be of flipping a Heads and rolling an even number on the die? (½ * ½ = ¼)

- Go back to the two coins. Suppose I was able to make an unfair coin which would come up Heads 75% of the time and I flip the coin twice.
- There are four possible outcomes but are they each equally likely to occur? Explain.
- What is the probability of getting Heads in both flips? (.75 * .75 = .5625)

- Another way to visualize this problem is using rectangle area. As the probability of flipping Heads increases, the area for the Heads/Heads event becomes larger. Note the two examples below. The events of H-H, H-T, T-H, and T-T each get a part of the probability "cake", which has a total area of 1. The area of their respective regions is their probability. One picture corresponds to the fair coin and the other to the unfair coin. Explain how the first and second flips and the probability of their outcomes are represented in the picture.





Shaquille O'Neal story:

Now it is time to go back and finish the freethrow story.

- What is the probability that Shaq makes both freethrows and the Heat win the game?
  .525*.525 = .275
- What is the probability that the Heat lose the game?
  .475*.475 = .500
- What is the probability that the game goes into overtime.
  .525*.475 + .475*.525 = .225
- What do all of these add up to? Shouldn't it be 1 or 100%? Most students will only calculate the Make/Miss option and will not consider that there are two possible outcomes (Make/Miss or Miss/Make).

The real conclusion to the story is that Shaq makes the first freethrow and misses the second, sending the game into overtime. The Jazz win in overtime!

Additional Samples:

- Johnny is a very accomplished archer and can hit the bulls-eye 82% of his shots.
  a. What is the probability that Johnny hits the bulls-eye twice in a row?
  .82*.82 = .6724
  b. What is the probability that Johnny hits at least one bulls-eye out of two shots?
  1-.18*.18 = .9676

- A bag of candy bars contains 5 Milky Way and 8 Snickers bars. Sam and Becky both love Milky Way candy bars and won't each a Snickers. What is the probability that Sam and Becky both pick a Milky Way bar out of the bag? (Assume that the first candy bar is not returned after being picked)
  5/13 * 4/12 = .1282

Extension:

Let's say that Shaq was fouled attempting a three point basket at the end of the game. This would mean that he gets to attempt three freethrows. What is the probability of making all three shots? What is the probability of missing all three shots? Discuss the other possible outcomes for this event and the probabilities of each.

1st shot	2nd shot	3rd shot		
Make	Make	Make	.525*.525*.525 =	0.14
Make	Make	Miss	.525*.525*.475 =	0.13
Make	Miss	Make	.525*.475*.525 =	0.13
Miss	Make	Make	.475*.525*.525 =	0.13
Make	Miss	Miss	.525*.475*.475 =	0.12
Miss	Make	Miss	.475*.525*.475 =	0.12
Miss	Miss	Make	.475*.475*.525 =	0.12
Miss	Miss	Miss	.475*.475*.475 =	0.11

P(make 3 shots) = 14%; P(make 2 of 3) = 39%; P(make 1of 3) = 36%; P(make 0 of 3) = 11%