

What does a walleye weigh?

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Abstract:

This lesson combines geometry and statistics to examine the relationship between scale factor and volume of similar figures using an estimation of walleye weights based upon the length of the fish. Students will incorporate scatterplots and regression equations along with investigating the scale factor/volume concept to construct a meaningful mathematical model.

Level & Standards: 10th grade; Geometry and Statistics

Time: 2 class periods required (90 minutes)

Materials: Manipulative cubes (substitute sugar cubes or dice) and/or isometric dot paper.

Objectives: Students will be able to

1. identify that the ratio of the volumes of similar objects is the cube of the ratio of the lengths (or scale factor)
2. apply that relationship to a real world model using statistics

Technology: Graphing calculators. (TI-83 or comparable)

Web Research Link to Montana Fish Wildlife and Parks:
<http://fwp.mt.gov/education/fishingeducation/walleye.html>

Downloadable File Links:
Walleye Student Page

Teacher Notes:

An important student prerequisite is the understanding that the ratio of volumes of two cubes is the cube of the length of each side. You may want to demonstrate with drawings or blocks.

Answers to Student Pages:

1. Students will probably think this graph looks like part of a parabola and will think of a quadratic equation.

2. Using a quadratic regression, $y = .0244x^2 - .5246x + 3.6587$ (rounded to nearest ten-thousandth).
3. Most students will see that the equation fits the data well by looking at the graph.
4. Make a table that relates the side and volume of these cubes.

i. Cubes

2. Length	3. 1	4. 2	5. 3	6. 4
7. Volume	8. 1	9. 8	10. 27	11. 64

(Students could make the models up to $3 \times 3 \times 3$ cube, but should calculate the $4 \times 4 \times 4$ cube)

5. The length of the side³ = volume. This is unique to cubes, but something similar holds for other figures.
6. Use this for a class discussion or possibly an assignment for students to research in a textbook or on the Internet. Similar figures are geometric figures where corresponding angles are congruent and corresponding sides are proportional (proportional meaning equal ratios). The idea of similarity is that of figures that have the same shape, but not necessarily the same size. Scale factor is the ratio of the lengths of corresponding sides of similar figures.
7. This could serve as a homework assignment. Sample answer: a $2 \times 3 \times 4$ rectangular prism has a volume of 24 units³; a $4 \times 6 \times 8$ rectangular prism has a volume of 192 units³—this is 8 times the volume of the smaller prism. The larger prism is twice as big in each direction and the volume is 8 times as much— $2^3 = 8$.
8. This could serve as a homework assignment. Sample answer: A cylinder with a radius of 2 units and height of 5 units has a volume of 62.83 units³; a cylinder with a radius of 6 units and a height of 15 units has a volume of 1696.46 units³. The larger cylinder is 3 times larger than the smaller cylinder and has 27 times the volume. $3^3 = 27$.
9. The ratio of the scale factors cubed is equal to the ratio of the volumes.
10. The ratio of the volumes is 1:64.
11. Check students' graphs. On the TI-83 the cubic regression $y = .0005x^3 - .0077x^2 + .1782x - 1.3415$ (rounded to the nearest ten-thousandth). This equation visually fits the data as well as the quadratic model used earlier. This model is a better one because of the cubic relationship between fish lengths (scale factor) and volume. This model predicts that a 31" walleye weighs 11.0 lbs. (to the nearest tenth).
12. Again check students work on the calculator. A Cubic regression $y = .0004x^3 - .0100x^2 + .3192x - 3.3508$ (rounded to the nearest ten-thousandth). Using this model a 24" pike weighs 3.9 lbs. and a 41" pike weighs 19.7 lbs. (to the nearest tenth). A 25 lb pike weighs about 44.3" from the model (use the table feature of the graphing calculator to obtain this value).
13. Volume is a function of the length cubed for similar figures. This concept can even be extended to the fish that populate the streams of Montana!